Myths about the Analysis of Change

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Myths about the Analysis of Change

Introduction

Education, psychology, and the social sciences in general are constantly engaged in study of the nature of change and the factors influencing it.

For example, understanding change is the fundamental goal of Developmental Psychology.

Introduction

The study of change is an extremely challenging topic in applied statistics. There have been numerous false starts and misconceptions. An important step forward was taken in 1988, when David Rogosa published his classic article *Myths about Longitudinal Research*, which he subsequently updated and expanded into a chapter in John Gottman's edited volume *The Analysis of Change.*

Today we review some of the key ideas in Rogosa's article.

Limitations of the Two-Observation Study

Many longitudinal studies have used only two measurements per individual—often in a Pretest-Posttest design.

Despite the popularity of two-wave designs, we must caution that, strictly speaking, two repeated measurements qualify as a longitudinal study, but only a very weak one.

Inability to Estimate Functional Form

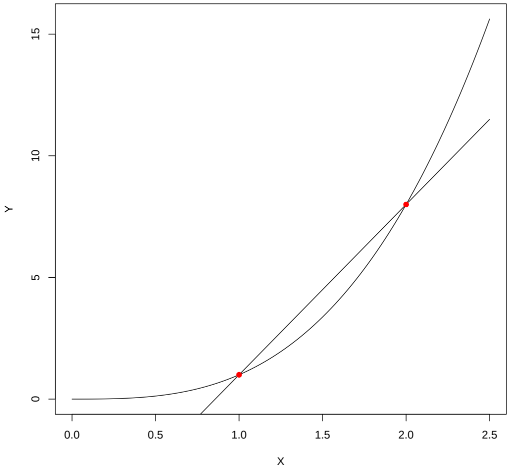
Two observations are insufficient to identify the form of a function.

An infinity of functions can be adjusted to pass through two data points.

Example

- Here is a graph of the points $(1, 1)$ and $(2, 8)$
- These points fit the functions $y = 7x - 6$ and $y = x^3$ perfectly
- Without an additional data point, we cannot discriminate empirically between the two functions

Inability to Estimate Functional Form



Amount of Change Will Often Be Deceptive

Rogosa (p. 10–11) considers a more ambitious example where 6 individuals have different growth curves from the same exponential family"

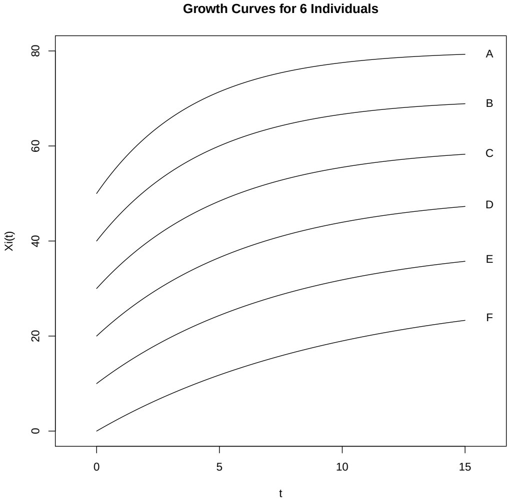
$$\xi_p(t) = \lambda_p - (\lambda_p - \gamma_p(0)) \exp(-\gamma_p t) \tag{1}$$

The growth curves (and the R code that generated them) are shown on the next two slides.

Growth Curves for 6 individuals

```
> lambda <- c(80,70,60,50,40,30)
> gamma <- c(.25,.22,.19,.16,.13,.1)
> a <- c(50,40,30,20,10,0)
> xi <- function(p,t)
+ {
+ lambda[p] - (lambda[p] - a[p])*exp(-gamma[p]*t)
+ }
> curve(xi(1,x),0,15,ylim=c(0,80),ylab="Xi(t)",xlab="t",mai
> curve(xi(2,x),0,15,add=T)
> curve(xi(3,x),0,15,add=T)
> curve(xi(4,x),0,15,add=T)
> curve(xi(5,x),0,15,add=T)
> curve(xi(6,x),0,15,add=T)
```

Growth Curves for 6 individuals



Growth Curves for 6 individuals

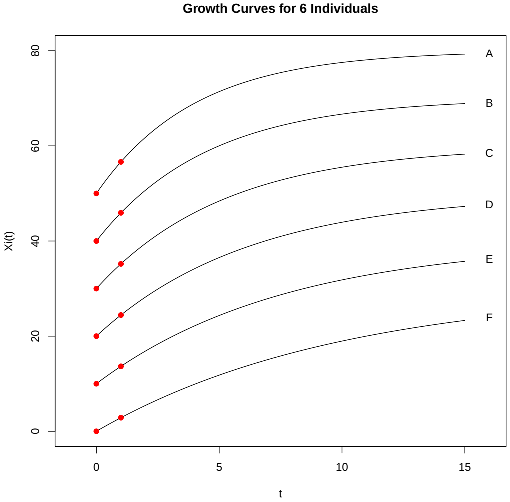
The curves vary obviously in their height, but far less obviously in their rates of change at various points.

If we load these curves into R, we can quickly see that the change between time t and time $t+1$ actually does vary substantially across the 6 individuals.

Even more interesting is the fact that the *rank order* of the change varies, depending on the initial measurement point t.

For example, suppose the first measurement is at $t = 0$ and the second at $t = 1$. As you can see from the red points on the plot, individual A, the top plot, has the most change, and individual F, the bottom plot, has the least.

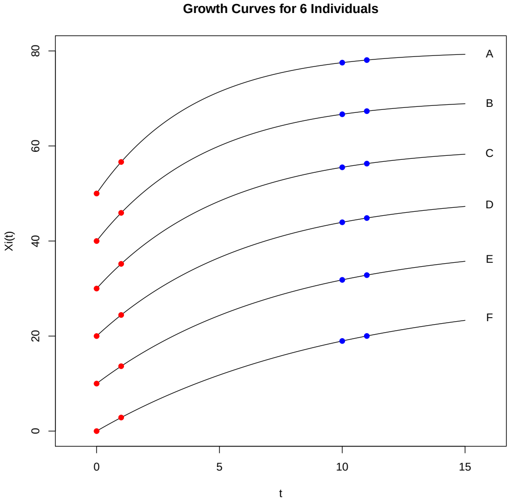
Growth Curves for 6 individuals



Growth Curves for 6 individuals

On the other hand, if we take the the first measurement is at $t = 10$ and the second at $t = 11$, as shown by the blue points, the rank ordering is reversed. Individual F has the most change, and individual A has the least.

Growth Curves for 6 individuals



Basic Properties of the Difference Score

Suppose you measure an individual at two time periods, and the observed values are realizations of the random variables X_1 and X_2.

As a natural measure of change, you take $D = X_2 - X_1$. What are some of the most important mathematical properties of D, the difference score?

Basic Properties of the Difference Score

To begin with, we need to recall the basic properties of variances, covariances, linear transformations and combinations.

If you have forgotten these, please go to the Psychology 310 Section and review these, or, if you are more advanced, go to the Psychology 312 website and read the chapter on the algebra of variances and covariances.

For now, we'll recall a couple of basic ideas.

Basic Properties of the Difference Score

Relevant Algebraic Results

Basic Covariance Algebra

- Adding a constant to a variable has no effect on variances, covariances, or correlations
- The correlation between X and Y is their covariance divided by the product of their standard deviations, i.e.,

$$\rho_{x,y} = \frac{\sigma_{x,y}}{\sigma_x \sigma_y} = \frac{\sigma_{x,y}}{\sqrt{\sigma_x^2 \sigma_y^2}}$$

- To compute the variance of a linear combination or transformation, (a) remove all constants from the expression, then (b) square the expression and apply the conversion rule:
    - Replace the square of any variable with the variance of that variable
    - Replace the product of any two variables with the covariance of those variables

Basic Properties of the Difference Score

Relevant Algebraic Results

Example (Variance of $X + Y$)

Square the expression:

$$(X + Y)^2 = X^2 + Y^2 + 2XY$$

Apply the conversion rule:

$$X^2 + Y^2 + 2XY \to \sigma_x^2 + \sigma_y^2 - 2\sigma_{x,y}$$

Basic Properties of the Difference Score

Relevant Algebraic Results

Example (Variance of $3X + 4$)

Remove the constant:

$$3X + 4 \rightarrow 3X$$

Square the expression:

$$(3X)^2 = 9X^2$$

Apply the conversion rule:

$$9X^2 \rightarrow 9\sigma_x^2$$

Basic Properties of the Difference Score
Relevant Algebraic Properties

A similar result holds for the *covariance* between two linear combinations or transformations.

Covariance of Two Linear Combinations

- Remove all constants from the expression for each linear combination, then
- Compute the product of the two expression
- Apply the same conversion rule that we used for variances

Basic Properties of the Difference Score

Relevant Algebraic Results

Example (Covariance of $X_1 = \xi + E_1$ and $X_2 = \xi + E_2$)

Suppose a random variable ξ has a variance of σ^2_ξ and random variables E_1 and E_2 are uncorrelated with each other and with ξ, i.e., they represent random error. Suppose that E_1 and E_2 both have variance σ^2_e. What will be the covariance between X_1 and X_2?

- Taking the product of the two expressions, we get

$$(\xi + E_1)(\xi + E_2) = \xi^2 + \xi E_1 + \xi E_2 + E_1 E_2$$

- Applying the conversion rule, we get

$$\sigma^2_\xi + \sigma_{\xi,e_1} + \sigma_{\xi,e_2} + \sigma_{e_1,e_2} = \sigma^2_\xi + 0 + 0 + 0 = \sigma^2_\xi$$

Basic Properties of the Difference Score

Relevant Algebraic Results

In the preceding slide, we derived a well-known result in classical test theory. If we assume that X_1 and X_2 are in standard score form (so that their covarariance is also their correlation), then the *test-retest correlation* measures the proportion of the variance of X that is due to ξ, the "true score." This is known as the reliability of X.

The above is an example of the kind of calculations we can routinely perform to examine the properties of observed scores and their differences.

Unreliability of the Difference Score

A result that appears in a number of places in the literature is the fact that, under certain conditions, if you take measurements X_1 and X_2, each of the X's will have reliability much greater than the difference score $D = X_1 - X_2$. In other words, even when the individual measurements have high reliability, the difference score may have low reliability. If the tests have equal reliability and equal variance, the formula is

$$\rho_D = \frac{\rho_x - \rho_{x_1,x_2}}{1 - \rho_{x_1,x_2}} \tag{2}$$

where ρ_D is the reliability of D and ρ_x is the reliability of X_1 and of X_2. We will investigate the algebra of this in a class handout and lab exercise.

Rogosa points out that this result depends in part on the assumptions involved. In particular, if the correlation between ξ_1 and ξ_2 is only moderate, then true score reliability may be substantially higher than that given by the above formula.

The Value of the Longitudinal Correlation Matrix

A common belief is that the longitudinal correlation matrix, i.e., the correlation matrix of the variables over time, can tell you "whether or not you are measuring the same thing over time."

The foundation for this belief is the axiom, stated numerous times in the literature, is that if the measure "changes out from under you," then assessment of change using the measure is impossible.

The Value of the Longitudinal Correlation Matrix

Where truth gives way to myth is in the belief that correlations between measures at time i and time j, if low, indicate that "something different is being measured." (or, conversely, that a high correlation indicates that the same thing is being measured).

Rogosa's example debunking the myth is straightforward. Suppose you have a collection of individuals whose growth curves intersect, i.e., have substantial differences in their slopes and intercepts. Then the ordering of individuals may be quite different at time i and time j, and therefore the longitudinal correlation may be quite low, even though the same thing is in fact being measured at both times.