

# Introducing the Multilevel Model for Change

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GCM, 2010

- 1 Multilevel Modeling - A Brief Introduction
- 2 Hierarchical Data Structures
- 3 A Not-So-Simple “Simple Example”
- 4 Fitting the Model with R
- 5 Fitting the Model with HLM
  - The HLM Program
  - Data Preparation
  - Constructing the MDM File
  - Outcome Variable Specification
  - Model Analysis

# Introduction

In this lecture, we introduce the *multilevel model for change*. We begin with a quick review of multilevel models, and then specialize to the multilevel growth curve model.

# Hierarchical Data Structures

In many situations in data analysis, the data are *nested*, which often implies a *hierarchical* structure. For example:

- soldiers are nested within platoons, which are nested within companies, etc.
- Students are nested within classes, which are nested within schools, which are nested within school districts, etc.
- Of special interest in our current application is how, in longitudinal data analysis, *observations can be nested within individuals*.

## A Not-So-Simple “Simple Example”

In his introductory text, Goldstein (1999) considers a sample of students from the Junior School Project (JSP) data.

- There were over 1000 students nested within 50 schools
- Each student took the Raven progressive matrices test (an ability measure) and also was assessed several times in English and Mathematics achievement.
- Suppose we consider, for the time being, just the relationship between the Raven test and the first test result in Mathematics.

## Notational Choices

There are many ways we can write a multilevel model. We'll look at several here. We'll start with a classic two-level approach used by Raudenbush and Bryk. Keep in mind that you have to process the *ideas*. The notational variations (use of different letters, etc.) that you will see in various books and articles are virtually endless.

## The Level-1 Model

At Level 1, we can analyze the relationship between the Raven test and Math Achievement within each school. In this case, if we fit a simple linear regression line to the data, the model can be written for the  $i$ th student in the  $j$ th school as

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + r_{ij} \quad (1)$$

The  $r_{ij}$  are regression residuals and are assumed to be independently and identically normally distributed with a mean of 0 and a variance of  $\sigma^2$ .

## The Level-2 Model

Notice how each school has its own slope and intercept. The slope and intercept define the regression line. These slopes and intercepts can be viewed as arising through random normal variation around central value. Specifically, the Level-2 model is

$$\beta_{0j} = \gamma_{00} + u_{0j} \quad (2)$$

$$\beta_{1j} = \gamma_{10} + u_{1j} \quad (3)$$

The  $\beta_{0j}$ s and  $\beta_{1j}$ s are allowed to correlate, so they have a covariance matrix  $\mathbf{T}$  that is not necessarily diagonal.

## The Combined Mixed Model

By simply substituting the Level-2 model into the Level-1 expression, we get the combined (mixed) model which contains both fixed (the  $\gamma$ s) and random (the  $\beta$ s) effects. We get

$$\begin{aligned} Y_{ij} &= \gamma_{00} + u_{0j} + (\gamma_{10} + u_{1j})X_{ij} + r_{ij} \\ &= \gamma_{00} + \gamma_{10}X_{ij} + (u_{0j} + u_{1j}X_{ij} + r_{ij}) \end{aligned} \quad (4)$$

$$= \gamma_{00} + \gamma_{10}X_{ij} + r_{ij}^* \quad (5)$$

$$= \gamma_{00} + \gamma_{10}X_{ij} + (u_{0j} + u_{1j}X_{ij}) + r_{ij} \quad (6)$$

The above *multilevel* model is a special case of the *linear mixed effects model*. The  $\gamma$ s are *fixed effects*, and the  $u$ s are *random effects*, i.e., random variables having a distribution.

## The Combined Model – Matrix Notation

As I already mentioned, we can express the above model in a wide variety of notations. Here is one matrix notation. First, let's simply convert the scalar notation directly into matrix notation using the same lettering. Let  $\mathbf{y}_j$  represent the vector of scores of the individuals in group  $j$ . Then

$$\mathbf{y}_j = \mathbf{X}_j\boldsymbol{\gamma} + \mathbf{X}_j\mathbf{u}_j + \mathbf{r}_j \quad (7)$$

Note that, in the above, the scores in  $\mathbf{X}_j$  can be either design codes or actual scores on observed variables, because of the longstanding tradition in regression modeling of treating observed scores on predictors as *fixed* regardless of whether they are actually fixed scores or random variables.

## The Combined Model – Matrix Notation

Suppose we have just two schools and there are 3 students in school 1 and 4 students in school 2. The notation corresponds to the data as follows:

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} \boldsymbol{\gamma} + \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} \\ Y_{21} \\ Y_{31} \\ \hline Y_{12} \\ Y_{22} \\ Y_{32} \\ Y_{42} \end{bmatrix} = \begin{bmatrix} 1 & X_{11} \\ 1 & X_{21} \\ 1 & X_{31} \\ \hline 1 & X_{12} \\ 1 & X_{22} \\ 1 & X_{32} \\ 1 & X_{42} \end{bmatrix} \begin{bmatrix} \gamma_{00} \\ \gamma_{10} \end{bmatrix} + \begin{bmatrix} 1 & X_{11} & 0 & 0 \\ 1 & X_{21} & 0 & 0 \\ 1 & X_{31} & 0 & 0 \\ \hline 0 & 0 & 1 & X_{12} \\ 0 & 0 & 1 & X_{22} \\ 0 & 0 & 1 & X_{32} \\ 0 & 0 & 1 & X_{42} \end{bmatrix} \begin{bmatrix} u_{01} \\ u_{11} \\ u_{02} \\ u_{12} \end{bmatrix} + \begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \\ \hline r_{12} \\ r_{22} \\ r_{32} \\ r_{42} \end{bmatrix}$$

## The General Linear Mixed-Effects Model

The preceding can be seen (with some minor changes in lettering) to be a special case of the General Linear Mixed-Effects Model. Pinheiro and Bates (2000) in their well-known book, *Mixed-Effects Models in S and S-Plus* use the following notation for the GLMM for a single level of grouping, i.e., what Raudenbush and Bryk refer to as a two-level model.

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{b}_i + \boldsymbol{\epsilon}_i \quad (8)$$

$$\mathbf{b}_1 \sim N(\mathbf{0}, \boldsymbol{\Psi}), \quad \boldsymbol{\epsilon}_i \sim N(\mathbf{0}, \sigma^2\boldsymbol{\Lambda}_i) \quad (9)$$

where  $\mathbf{X}_i$  is the fixed effects regressor matrix for the  $i$ th unit, and  $\mathbf{Z}_i$  is the random effects regressor matrix, which usually contains a subset (perhaps all) of the columns of  $\mathbf{X}_i$ . The vector  $\boldsymbol{\beta}$  contains fixed effects, while  $\mathbf{b}_i$  contains the random effects.

## Fitting the Two-Level Model

The two level model with random slopes and intercepts can be fit with two rather different programs.

- One program, R, is free, and very general.
- The other, HLM, is a well-known commercial program designed to fit hierarchical linear models.
- R requires that you compute the full mixed model, as shown in Equation 6.
- HLM allows you to input the Level-1 and Level-2 models separately, and declare whether the effects for each are fixed or random.

To get a feel for how they work, we'll try both programs on the high school data.

## Fitting the Model with R

To fit LME models in R, we need to use the `lmer` function in the `lme4` package. We begin by loading in the data file, which is in long format. We'll convert it to wide format and remove missing data.

```
> library(foreign)
> jsp.long <- read.table("jsp.csv", header=TRUE,
+ sep = ",")
> jsp.wide <- reshape(data=jsp.long,direction="wide",
+ timevar="year",idvar="pupil")
> jsp.wide <- na.omit(jsp.wide)
> write.table(jsp.wide,"jsp.wide.txt",row.names=F,
+ col.names=T,sep=",")
```

# Fitting the Model with R

Then, we enter the model, using the appropriate syntax.

```
> full.model <- lmer(maths.0 ~ ravens.0 +  
+ (1 + ravens.0 | school.0),data=jsp.wide)  
> full.model
```

Linear mixed model fit by REML

Formula: maths.0 ~ ravens.0 + (1 + ravens.0 | school.0)

Data: jsp.wide

AIC BIC logLik deviance REMLdev

5618 5647 -2803 5601 5606

Random effects:

Groups	Name	Variance	Std.Dev.	Corr
school.0	(Intercept)	19.6206	4.430	
	ravens.0	0.0277	0.166	-0.935

Residual 30.2870 5.503

Number of obs: 887, groups: school.0, 48

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	7.9306	1.1241	7.06
ravens.0	0.6917	0.0428	16.14

Correlation of Fixed Effects:

(Intr)  
ravens.0 -0.962

## Fitting the Model with HLM

With HLM, rather than fitting the mixed model directly, we input the Level-1 and Level-2 models separately. This is a rather straightforward (if somewhat idiosyncratic) process.

## The HLM Program

HLM is a popular software program that makes construction of basic multilevel models relatively straightforward. In particular, it does not require combination of models from two or more levels into a single regression model. Consequently, many find it very convenient and (relatively) easy to use, which has contributed to its popularity. In this introduction, we will revisit the model that we examined, and set it up and analyze it in HLM.

We assume that you have the HLM6 program (full or student version) installed on your computer.

## Data Preparation and Input

HLM has limited (and somewhat disguised) data input capabilities. In practice, you will probably input most of your data as either SPSS *.sav* files (if you have the Student Version), or comma-delimited ASCII files with a header containing column names (if you have the full version). Since R writes *ascii* files routinely using the `write.table()` function (and the `sep = ','` option), and also has extensive data manipulation capabilities, you may find it convenient to use R to construct your HLM files.

## Data Preparation and Input

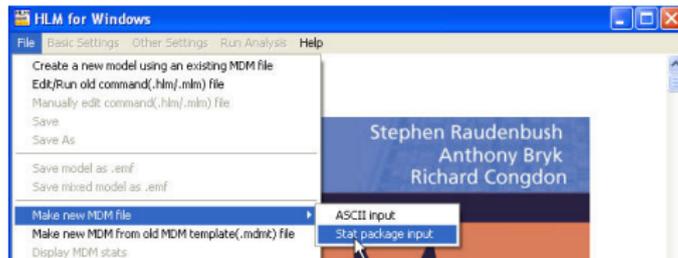
The link between the level-1 and level-2 models in the HLM parameterization is the subscript  $j$ , which refers in the current example to the `school.0` variable. To set up the data for HLM, we need two files, one for the level-1 variables, one for the level-2 variables. Each file has to be sorted in ascending order of the ID variable.

We need to include `ravens.0`, `math.0`, and `school.0` in the level-1 file, and `school.0` in the level-2 file. Since all the variables we need are in one file, we are ready to go.

# Constructing the MDM File

Setting up the Multivariate Data Matrix (MDM) file is a key first step to using HLM2 to analyze a 2-level model problem.

Make sure you have the file `jsp.wide.txt` available, or, if you are using the Student Version, the SPSS file `jsp.wide.sav`. We will show the procedure for the SPSS file. Begin by starting up HLM. Then click on the *Make New MDM File -> Stat Package Input* menu option, whether you are loading an SPSS file or a comma-delimited text file! (This is counterintuitive and very poor human factors design. One would expect this to be found under the ASCII file input node.)

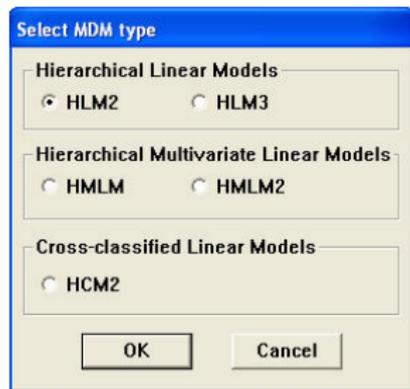


## Constructing the MDM File

Next, you will be asked to select a program.

Select HLM2.

Then click on the OK button.



## Constructing the MDM File

A large dialog box will open. Go to the drop-down list for file type, and select *SPSS/Windows*.

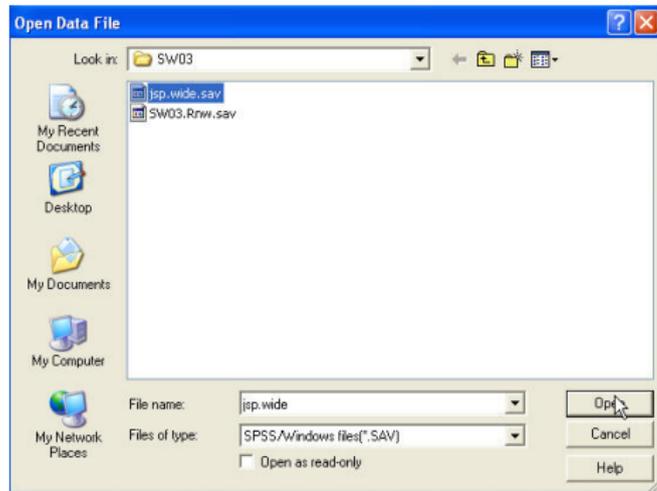
The screenshot shows the 'Make MDM - HLM2' dialog box. The title bar is blue with the text 'Make MDM - HLM2'. The dialog is divided into several sections:

- MDM template file:** Includes a 'File Name:' text box and three buttons: 'Open mdmt file', 'Save mdmt file', and 'Edit mdmt file'.
- MDM File Name (use .mdm suffix):** A text box for the file name.
- Input File Type:** A dropdown menu currently showing 'SPSS/Windows'.
- Nesting of input data:** Two radio buttons: 'persons within groups' (selected) and 'measures within persons'.
- Level-1 Specification:** Includes a 'Browse' button, a 'Level-1 File Name:' text box, and a 'Choose Variables' button.
- Missing Data?:** Two radio buttons: 'No' (selected) and 'Yes'. To the right, 'Delete missing data when:' has two radio buttons: 'making mdm' and 'running analyses'.
- Level-2 Specification:** Includes a 'Browse' button, a 'Level-2 File Name:' text box, and a 'Choose Variables' button.
- Bottom buttons:** 'Make MDM', 'Check Stats', and 'Done'.

## Constructing the MDM File

If you are not already there, go to the directory where the data files are.

Select *jsp.wide.sav* and click on the *Open* button.



## Constructing the MDM File

Look in the dialog box for the grouping that is titled *Level-1 Specification*. I’ve highlighted the group in red in the picture below.

On the right side of that grouping is a button *Choose Variables*.

The screenshot shows the 'Make MDM - HLM2' dialog box. The 'Level-1 Specification' section is highlighted in red. It contains a 'Browse' button, a text field for 'Level-1 File Name' (D:\I\Current Projects\IGCML\lectures\SW03\sp.wid), and a 'Choose Variables' button. Below it are 'Missing Data?' options: 'No' (selected), 'Yes', 'making mdm', and 'running analyses'. The 'Level-2 Specification' section also has a 'Browse' button, a text field for 'Level-2 File Name' (D:\I\Current Projects\IGCML\lectures\SW03\sp.wid), and a 'Choose Variables' button. At the bottom are 'Make MDM', 'Check Stats', and 'Done' buttons.

Click on it.

# Constructing the MDM File

A dialog box will open up that will allow you to select and classify level-1 variables. The variable that spans the two levels of your model is `SCHOOL.0`, and this variable is designated an ID variable. The variables `MATHS.0` and `RAVENS.0` are in the level-1 model, so they are checked off as being in the MDM. When you are ready to exit the dialog, it should look like this:

Choose variables - HLM2

PUPIL	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM	RAVENS.1	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
SCHOOL.0	<input checked="" type="checkbox"/>	ID	<input type="checkbox"/>	in MDM	ENGLISH1	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
CLASS.0	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM	MATHS.1	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
SEX.0	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM	SCHOOL.2	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
SC.0	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM	CLASS.2	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
RAVENS.0	<input type="checkbox"/>	ID	<input checked="" type="checkbox"/>	in MDM	SEX.2	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
ENGLISH0	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM	SC.2	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
MATHS.0	<input type="checkbox"/>	ID	<input checked="" type="checkbox"/>	in MDM	RAVENS.2	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
SCHOOL.1	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM	ENGLISH2	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
CLASS.1	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM	MATHS.2	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
SEX.1	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM		<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM
SC.1	<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM		<input type="checkbox"/>	ID	<input type="checkbox"/>	in MDM

Page 1 of 1

OK Cancel

## Constructing the MDM File

Next, go to the *Level-2 Specification* group and first, click on the *Browse* button. Again select the file *jsp.wide.sav*. Second, click on the *Choose Variables* button.

The screenshot shows the 'Make MDM - HLM2' dialog box. The 'Level-2 Specification' section is highlighted with a red box, and red arrows point to the 'Browse' and 'Choose Variables' buttons. The dialog box contains the following elements:

- MDM template file:** File Name: [text box], Open mdmt file, Save mdmt file, Edit mdmt file, MDM File Name (use .mdm suffix): [text box], Input File Type: SPSSWindows (dropdown).
- Nesting of input data:**  persons within groups,  measures within persons.
- Level-1 Specification:** Browse, Level-1 File Name: D:\!!!Current Projects\!GCM\Lectures\!SW03\!jsp.wid, Choose Variables, Missing Data?  No,  Yes, Delete missing data when:  making mdm,  running analyses.
- Level-2 Specification:** Browse, Level-2 File Name: [text box], Choose Variables.
- Buttons:** Make MDM, Check Stats, Done.

## Constructing the MDM File

This will take you to another variable selection dialog. Again `school` is the ID variable, and we actually have no level-2 predictors in this model. However, HLM requires you to select at least one additional variable, so choose the same variables as for the level-1 file.

## Constructing the MDM File

At this point, the HLM program again exhibits poor human factors design. The necessary next step is to save a mdmt “template file.” However in order to do that, you have to enter the name of the mdm file you want to save! You enter *jsp.mdm* in the appropriate edit field, then click on *Save mdmt file*.

The screenshot shows the 'Make MDM - HLM2' dialog box. It is divided into several sections:

- MDM template file:** Contains a 'File Name:' field with 'jsp.mdm' entered. Below it are three buttons: 'Open mdmt file', 'Save mdmt file', and 'Edit mdmt file'. To the right is an 'MDM File Name (use .mdm suffix)' field, also containing 'jsp.mdm', and an 'Input File Type' dropdown menu set to 'SPSSAWindows'.
- Nesting of input data:** A section with two radio buttons: 'persons within groups' (selected) and 'measures within persons'.
- Level-1 Specification:** Includes a 'Browse' button, a 'Level-1 File Name:' field with the path 'D:\Current Projects\IGCM\Lectures\SW03\jsp.wid', and a 'Choose Variables' button.
- Missing Data?:** A section with two radio buttons for 'Delete missing data when:': 'No' (selected) and 'Yes'. Below these are two more radio buttons: 'making mdm' and 'running analyses'.
- Level-2 Specification:** Includes a 'Browse' button, a 'Level-2 File Name:' field, and a 'Choose Variables' button.
- Bottom:** Three buttons: 'Make MDM', 'Check Stats', and 'Done'.

## Constructing the MDM File

Once you've done saved the mdmt file, you can make the MDM file by clicking on the *make MDM* file at the bottom left.

Make MDM - HLM2

MDM template file

File Name:

MDM File Name (use .mdm suffix)

Input File Type

Nesting of input data

persons within groups  measures within persons

Level-1 Specification

Level-1 File Name:

Missing Data?  No  Yes

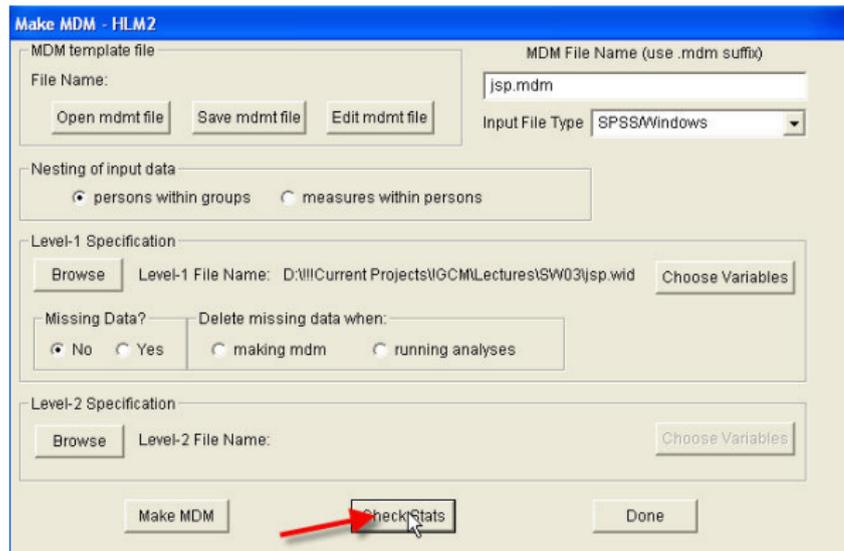
Delete missing data when:  making mdm  running analyses

Level-2 Specification

Level-2 File Name:

## Constructing the MDM File

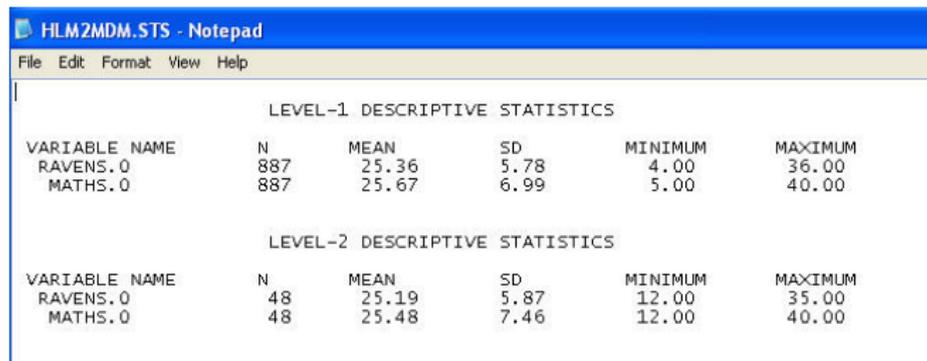
At this point, you are strongly advised to Examine the basic statistics for the MDM file you have just created. You do this by clicking on the *Check Stats* button as shown below.



## Constructing the MDM File

Examine the statistics, see if you have chosen the correct variables, and check whether the descriptive statistics make sense.

Then click *Done*



The screenshot shows a Notepad window with the following content:

```
HLM2MDM.STS - Notepad
File Edit Format View Help

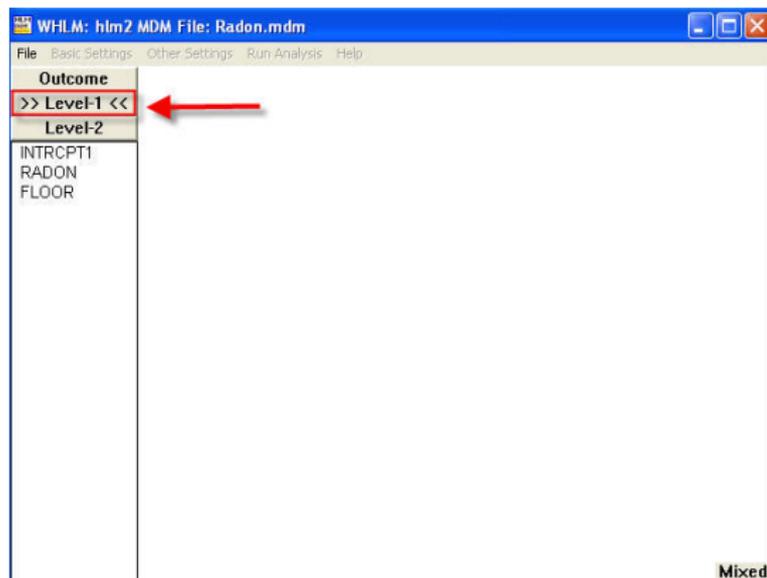
LEVEL-1 DESCRIPTIVE STATISTICS
VARIABLE NAME      N      MEAN      SD      MINIMUM      MAXIMUM
RAVENS.0            887     25.36     5.78     4.00         36.00
MATHS.0             887     25.67     6.99     5.00         40.00

LEVEL-2 DESCRIPTIVE STATISTICS
VARIABLE NAME      N      MEAN      SD      MINIMUM      MAXIMUM
RAVENS.0            48     25.19     5.87     12.00        35.00
MATHS.0             48     25.48     7.46     12.00        40.00
```

## Level-1 Specification

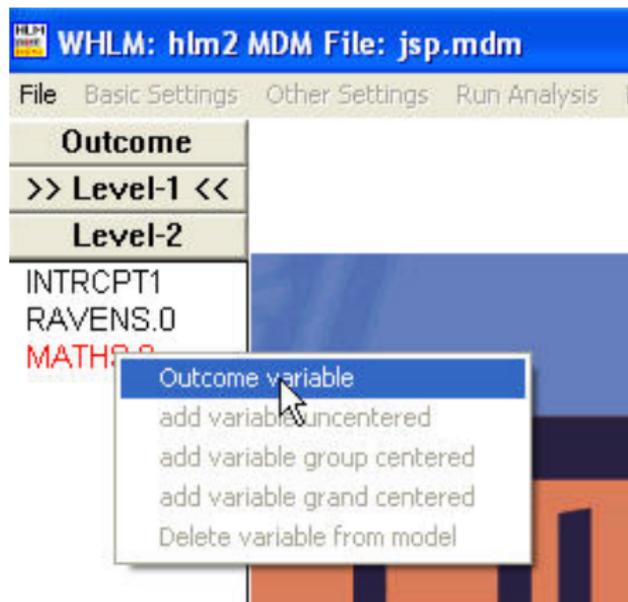
The next step is to specify the model.

Note that the Level-1 Button is highlighted.



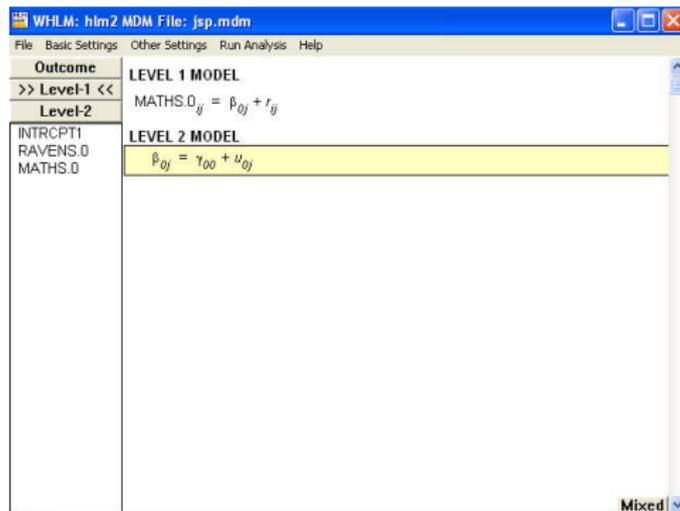
## Level-1 Specification

our first step is to choose the Level-1 outcome variable. Click on *MATHS.0* and a flying menu will open. Choose *Outcome variable*. You have now selected your outcome variable.



# Constructing the MDM File

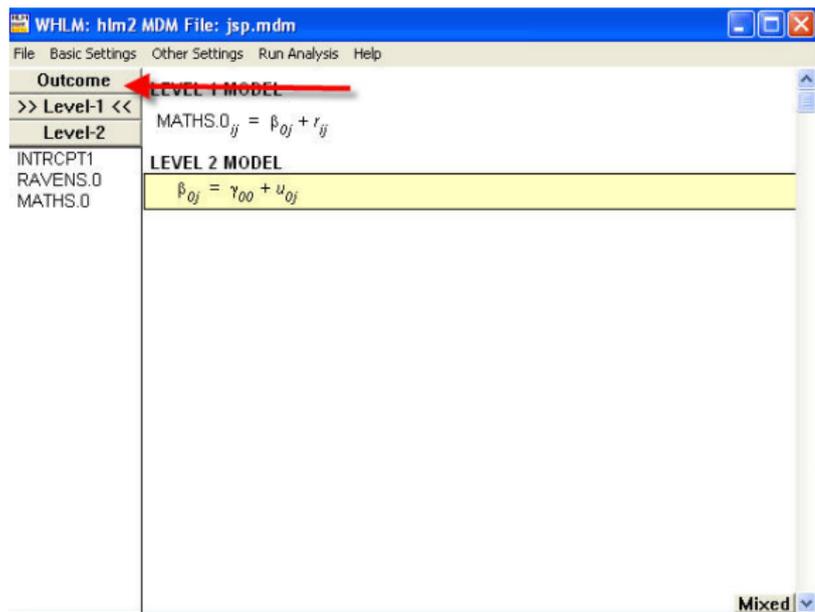
Next, you will see a window open up. This window will contain the current model specification in HLM notation. Note that a baseline Level-2 model has already been specified. Normally, you would next enter the Level-2 specification, but in this case, we are actually finished.



The screenshot shows a software window titled "WHLM: hlm2 MDM File: jsp.mdm". The window has a menu bar with "File", "Basic Settings", "Other Settings", "Run Analysis", and "Help". On the left side, there is a tree view with "Outcome" selected, and sub-items for "Level-1" and "Level-2". The "Level-1" section is expanded, showing "MATHS.0" with the equation  $MATHS.0_{ij} = \beta_{0j} + r_{ij}$ . The "Level-2" section is also expanded, showing "INTRCPT1", "RAVENS.0", and "MATHS.0" with the equation  $\beta_{0j} = \gamma_{00} + u_{0j}$ . The window title bar includes standard Windows window controls (minimize, maximize, close).

## Outcome Variable Specifications

The next step is to specify the characteristics of the outcome variable. Click on the *Outcome* button.



# Outcome Variable Specification

You'll see the dialog pictured below. Because we're assuming a normally distributed outcome, you don't have to do anything, although, if you wish, you could save residual files for analysis by another program.

Just click *Ok*. Note: if you don't do this, your model will not be specified! Most modern software assumes a default (in this case a normal outcome variable) but HLM does not.

Basic Model Specifications - HLM2

Distribution of Outcome Variable

Normal (Continuous)

Bernoulli (0 or 1)

Poisson (constant exposure)

Binomial (number of trials)

Poisson (variable exposure)

Multinomial Number of categories

Ordinal

Over dispersion

Title

Output file name

Graph file name

## Adding a Level-1 Predictor

In this case, we wish to predict MATHS.0 from RAVENS.0 at level 1. So we click on the variable name and add it, uncentered. Notice how the model is changed automatically to reflect the new predictor.

The screenshot shows the WHLM software interface. The title bar reads "WHLM: hlm2 MDM File: jsp.mdm". The menu bar includes "File", "Basic Settings", "Other Settings", "Run Analysis", and "Help". The "Outcome" section is expanded to show "Level-1" and "Level-2". The Level-1 model is currently defined as  $MATHS.0_{ij} = \beta_{0j} + r_{ij}$ . A context menu is open over the variable name "RAVENS.0" in the Level-1 list, with the option "add variable uncentered" selected. The Level-2 model section is also visible, showing "INTRCPT1" and "MATHS.0".

## Adding a Level-2 Random Term

The level-2 model includes random slopes and intercepts. Notice how the random effect  $u_{1j}$  is greyed out in the level-2 model. Click on it and it is immediately added to the level-2 model.

# Constructing the MDM File

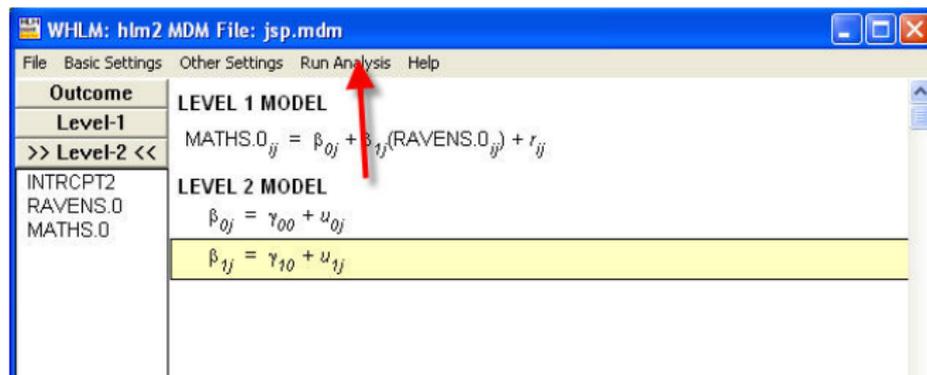
If you wish, HLM will automatically combine the two models into a single *mixed model*, which might be especially useful if you wish to use another program (like R) to analyze the model. Simply click on the *Mixed* button in the lower right corner of the main window.

In this case we can see that this model is indeed the same as what we had before.

The screenshot shows the HLM software interface. The main window is titled "WELM: hlm2 MDM File: jsp.mdm". It has a menu bar with "File", "Basic Settings", "Other Settings", "Run Analysis", and "Help". The main area is divided into two panes. The left pane shows a tree view with "Outcome" expanded to "Level-1" and "Level-2". The right pane shows the model equations for Level 1 and Level 2. The Level 1 model is  $MATHS.D_y = \beta_{0y} + \beta_{1y}(RAVENS.D_g) + r_{iy}$ . The Level 2 model is  $\beta_{0y} = \gamma_{00} + u_{0y}$  and  $\beta_{1y} = \gamma_{10} + u_{1y}$ . A "Mixed" button is visible in the bottom right corner of the main window. Below the main window is a smaller window titled "Mixed Model" which displays the combined mixed model equation:  $MATHS.D_y = \gamma_{00} + \gamma_{10} * RAVENS.D_g + u_{0y} + u_{1y} * RAVENS.D_g + r_{iy}$ .

## Analyzing the Model

Click on the *Run Analysis* button. HLM will open a DOS window and the model will run. There will be a brief pause near the end of calculations. Don't interrupt! The window will eventually shut.



The screenshot shows the WHLM software interface with the title bar "WHLM: hlm2 MDM File: jsp.mdm". The menu bar includes "File", "Basic Settings", "Other Settings", "Run Analysis", and "Help". A red arrow points to the "Run Analysis" menu item. The main window displays the model structure:

Outcome	LEVEL 1 MODEL
Level-1	$\text{MATHS.0}_{ij} = \beta_{0j} + \beta_{1j}(\text{RAVENS.0}_{ij}) + r_{ij}$
>> Level-2 <<	LEVEL 2 MODEL
INTRCPT2	$\beta_{0j} = \gamma_{00} + u_{0j}$
RAVENS.0	$\beta_{1j} = \gamma_{10} + u_{1j}$
MATHS.0	

## Viewing Output

The output window will not open automatically. You need to select the *File*→*View Output* menu option.

